Problem 04: n-th Power

Calculate the nth power of x.

$$\begin{array}{lcccccccc} A & = & \mathbb{Z} & \times & \mathbb{N}_0 & \times & \mathbb{Z} & |\times & \mathbb{N}_0 \\ & & x & & n & & y & & i \\ B & = & \mathbb{Z} & \times & \mathbb{N}_0 & & & & \\ & & x' & & n' & & & & \\ Q & = & (x' = x) \wedge (n' = n) & & & & & & \\ R & = & Q \wedge (y = x^n) & & & & & & \end{array}$$

Solution

We can construct the following loop (reached via $Q' = Q \wedge (i = n) \wedge (y = 1)$):

$$\begin{array}{lcl} P & = & Q \wedge i \in [0, n] \wedge (y = x^{(n-i)}) \\ \pi & = & (i \neq 0) \\ t & = & i \end{array}$$

As always, we solve P for decreasing t, i.e. for $i := i - 1$, using the fact that $x^{n+1} = x * x^n$:

$$P^{i \leftarrow i-1} \quad = \quad Q \wedge (i - 1) \in [0, n] \wedge (y = x^{(n-i)+1} = x * x^{(n-i)})$$

Thus arriving at the following program:

$y, i := 1, n$	
$i \neq 0$	
	$y := x * y$
	$i := i - 1$