Problem 24: Number of Digits

Given two natural numbers x and k, compute the number of digits needed to represent x in base-k.

$$
\begin{array}{rcccccccc}
A & = & \mathbb{N} & \times & \mathbb{N} & \times & \mathbb{N} & |\times & \mathbb{N} \\
 & & x & & k & & d & & y \\
B & = & \mathbb{N} & \times & \mathbb{N} & & & & \\
 & & x' & & k' & & & & \\
Q & = & \multicolumn{7}{l}{(x' = x) \wedge (k' = k)} \\
R & = & \multicolumn{7}{l}{Q \wedge k^d < x \leq k^{d-1}}
\end{array}
$$

Solution

We can use the same method as in problem 05, this time using integer division instead of subtraction. y initially stores a copy of x, and is only used because otherwise x would be changed (and thus the Q part of R wouldn't be satisfied).

$$
\begin{array}{lcl}
P & = & Q \wedge (y * k^{d-1} \leq x) \\
\neg\pi & = & y < k \\
\pi & = & y \geq k \\
t & = & \lfloor \log_k x \rfloor - d \\
Q' & = & Q \wedge (y = x) \wedge (d = 1) \\
 & & \\
P^{d \leftarrow (d+1)} & = & Q \wedge (y * k^d \leq x) \\
 & \simeq & P \wedge \pi \wedge (y = y \operatorname{div} k)
\end{array}
$$

The resulting program:

<table>
<tr><td colspan="2">$y, d := x, 1$</td></tr>
<tr><td colspan="2">$y \geq k$</td></tr>
<tr><td rowspan="2"></td><td>$y := y \operatorname{div} k$</td></tr>
<tr><td>$d := d + 1$</td></tr>
</table>